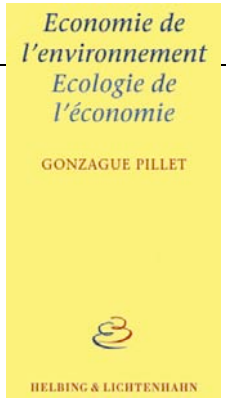

LECTURE OBJECTIVES
The object of this lecture is to give students a full understanding of the economic definition of externalities. Particular emphasis will be put on their role in the bargaining process. The second part of the lecture will focus on the victim-pays-principle and its mathematical derivation.

2006 BOOK IN FRENCH BY GONZAGUE PILLET | ENGLISH VERSION BY KAREN MAYOR

1 | EXTERNALITIES: DEFINITION AND MATHEMATICAL DERIVATION

THE CONCEPT OF EXTERNALITIES

Markets are a process by which scarce resources are allocated to their most valued uses. The majority of environmental goods are not market goods and it is hence not possible to directly introduce the environment into a cost or utility function. The theoretical link between the economy and the environment is the concept of **externalities**.

An externality or external effect exists when the consumption or production choices of one person or firm enters the utility or production function of another entity without that entity's permission or compensation.

The consumption of certain goods might result in negative environmental effects. For example, those commonly associated with road transport are air and noise pollution as well as congestion. It is important to dissociate distributional externalities from real income externalities. A distributional externality or **pecuniary externality** occurs when prices change because of a change in demand and consequently an agent's income is indirectly affected by the actions of others. We will not be concerned with this type of external effect. Real income or **technological externalities** occur when there is a direct effect through utility or production. These are the true diseconomies or external effects.

It is worth noting that external effects can be **positive**. The classic example is that of the bee keeper and the nearby orchard. The bees help pollinate the blossoms in the orchard and this generates a positive externality. Hence, externalities are an interrelationship between utility or production functions that is *external* to the market. It is this interrelationship that allows environmental impacts to be expressed in economic terms.

In summary, an external diseconomy exists when the following two conditions are satisfied:

- the activities of one agent lead to a loss of welfare for another agent;
- this loss in welfare is uncompensated.

There are then three consequences:

- the existence of "physical" pollution does not necessarily mean an "economic" pollution exists;
- even if an "economic" pollution exists this does not automatically mean that the corresponding "physical" pollution should be eliminated;

- there is a possible gain to be made through negotiation.

Intuitive results:

- economists do not recommend that the level of externalities be nil as intuitively, an optimal externality is not nil (it reflects the level of optimal pollution abatement, i.e. not complete pollution abatement);
- only the non-optimal share of external effects is hence targeted. The optimal share of pollution is the part of the external effect that is Pareto relevant, i.e. its elimination results in a net gain for society;
- negotiation is a potential solution to obtaining this net gain.

The next section looks at how to capture these externalities in **mathematical terms**.

MATHEMATICAL DEFINITION OF AN EXTERNALITY

Consider a market with two consumers i and j and n goods. Consumption of these goods affects the consumers' utilities and this is shown through their **utility functions**. For instance, agent i's utility is shown as follows:

$$U_i = u_i(x_{i1}, x_{i2}, ..., x_{in}) \qquad \textit{Equation 1}$$

Both agents have a budget constraint whereby their revenue must be higher or equal to their consumption.

$$\bar{Y}_i \geq \sum_{k=1}^{n} p_k x_{ik} \qquad \textit{Equation 2}$$

Where Y_i is the revenue for agent i and p_k is the price of a good within a choice set of n goods. There is then an external effect from i to j if:

$$\max!\ U_i = u_i(x_{i1}, x_{i2}, ..., x_{in}),\ \text{subject to} \sum_{k=1}^{n} p_k x_{ik} \leq \bar{Y}_i \qquad \textit{Equation 3}$$

$$\max!\ U_j = u_j(x_{j1}, x_{j2}, ..., x_{jn}, x_{1i}),\ \text{subject to} \sum_{k=1}^{n} p_k x_{jk} \leq \bar{Y}_j \qquad \textit{Equation 4}$$

i.e. if agent i's utility depends on goods that are under his control and agent j's utility depends on goods 1 to n that he has control over, but also good 1 which is consumed by i. Then, agent j's utility depends on effects that are outside the market and there is no price for them. In effect, the influence of x_{i1} does not appear in agent j's budget constraint.

Consequently, an external effect exists if the following equation holds, i.e. when the marginal utility for j of the consumption of good 1 by agent i is not nil. If equation 5 is positive there is an **external economy** if it is negative there is an **external diseconomy**.

$$\frac{\delta U_j}{\delta x_{i1}} \neq 0 \qquad \textit{Equation 5}$$

DERIVING THE OPTIMUM

When in the presence of an external diseconomy, it is agent j that has an incentive to take action in order to improve his welfare, without correspondingly making agent i worse off. In order to show this mathematically, we must use equations 3 and 4.

First, set up a Lagrangian and maximise U_i and U_j with respect to x_i:

$$\Lambda = f(x_{i1},...,x_{in}) - \lambda_i(p_k x_{ik} - \overline{Y}) \qquad \textit{Equation 6}$$

$$f_i^{'}(x_{ik}) \Rightarrow \frac{1}{\lambda_i}\frac{\delta U_i}{\delta x_{ik}} - p_k = 0 \qquad \textit{Equation 7}$$

$$f_i^{''} < 0$$

and then to x_j:

$$\Lambda = f(x_{j1},...,x_{jn};x_{i1}) - \lambda_j(p_k x_{jk} - \overline{Y}_j) \qquad \textit{Equation 8}$$

2 | THE VICTIM-PAYS-PRINCIPLE: CONCEPT AND DERIVATION

ASSUMPTIONS OF THE MODEL

As we have shown, when dealing with a negative external effect, it is agent j that is motivated to take action in order to reach an optimal solution. Interestingly, in this scenario, agent j is the **victim** of pollution.

Consider a model with the following assumptions:

- there are a small number of agents involved and they can communicate with each other;
- the transaction costs of negotiation do not exceed the potential gains to be made from negotiation;
- the analysis is limited to the situation with technological externalities;
- agents are free to make their own choices whether they are market choices or not;
- there are no legal restrictions or regulations that could potentially constrain a polluter's choices (i.e. there are no bans on polluting).

ALGEBRAIC DERIVATION

Once again consider a market with two agents i and j. Agent i causes a diseconomy to agent j through his consumption. Also, recall the agents' utility functions (equation 1) and their budget constraints (equation 2). Each agent maximises his utility subject to his budget constraint. We then get:

$$\frac{1}{\lambda_r}\frac{\delta U_r}{\delta x_{rk}}\bigg|_{x_{rk}} - p_k \begin{Bmatrix} \leq \\ = \end{Bmatrix} 0 \qquad \textit{Equation 9}$$

$$\text{for } \left| x_{rk} \begin{Bmatrix} = \\ > \end{Bmatrix} 0 \text{ for } k = 1,...n;\ r = i,j \right.$$

The lambdas are called the Lagrangians multipliers and represent the marginal utility of income for i and j. This equation then shows that the marginal utility of the k^e good, which is measured by the marginal utility of income, must be smaller or equal to the price of that good if the latter is not purchased and must be equal to the price of that good if it is purchased.

There is then an external diseconomy when:

$$\left.\frac{\delta U_j}{\delta x_{i1}}\right|_{x_{i1}} \neq 0 \qquad \textit{Equation 10}$$

Hence, when agents are in the presence of an external diseconomy and agent j is motivated to take action in order to reach an optimum; this external diseconomy is **Pareto relevant**. That is, the internalisation of the diseconomy can lead j to a higher level of welfare without negatively affecting agent i's welfare. So, how does agent j go about improving his welfare? This is done through **negotiation or bargaining** with agent i. Agent j (who bear in mind is the victim) will offer to pay i a certain amount ρ_e for each unit of consumption that agent i is willing to forego. This is how diseconomies are **internalised** through negotiation. Making these monetary offers will inevitably modify the maximisation conditions of the utility functions for both consumers. Indeed, their revenues will change:

$$\sum_{k=1}^{n} p_k x_{jk} \leq \overline{Y}_j - \rho_e(|x_{i1} - |x'_{i1}) \qquad \textit{Equation 11}$$

$$\sum_{k=1}^{n} p_k x_{ik} \leq \overline{Y}_i + \rho_e(|x_{i1} - |x'_{i1}) \qquad \textit{Equation 12}$$

This will accordingly alter their budget constraints:

For j:

$$\Lambda = f(x_{j1},...,x_{jn}) - \lambda_j(p_k x_{jk} - \overline{Y}_j + P_e) \qquad \textit{Equation 13}$$

$$\text{for } P_e = \rho_e(|x_{i1} - |x^*_{i1})$$

For i:

$$\Lambda = f(x_{i1},...,x_{in}) - \lambda_i(p_k x_{ik} - \overline{Y}_i - P_e) \qquad \textit{Equation 14}$$

And finally the conditions at optimum and hence the maximisation results must also be adjusted:

For j regarding current goods x_{jk}:

$$\frac{1}{\lambda^*_j}\left.\frac{\delta U_j}{\delta x_{jk}}\right|_{x^*_{jk}} - p_k \begin{Bmatrix} \leq \\ 0 \end{Bmatrix} 0 \qquad \textit{Equation 15}$$

$$\text{for } |x^*_{jk} \begin{Bmatrix} = \\ > \end{Bmatrix} 0$$

For j regarding environmental goods x_{i1}:

$$\frac{1}{\lambda^*_j}\left.\frac{\delta U_j}{\delta x_{i1}}\right|_{x^*_{i1}} - \rho_e \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} 0 \qquad \textit{Equation 16}$$

The negotiation process is iterative, that is agent j will make successive offers until the optimal ρ is attained and consequently an optimal level of x_{i1} is reached. If these conditions are met, there is no other situation whereby either agent can improve his or her welfare without affecting the welfare of the other agent; a **Pareto optimum** has been reached.

RESULTS

It has been shown that in the absence of any type of pollution regulation, it is the victim of the pollution who will have an incentive to increase his welfare through negotiation. In a situation where property rights are held by polluters, the victims are the ones that will be willing to pay for an increase in environmental quality.

$$\frac{1}{\lambda_j^*}\frac{\delta U_j}{\delta x_{jk}}\bigg|_{x_{jk}^*} - p_k = 0 \text{ for } \left|x_{jk}^* > 0\right. \qquad \textit{Equation 17}$$

$$\frac{1}{\lambda_j^*}\frac{\delta U_j}{\delta x_{i1}}\bigg|_{x_{i1}^*} - \rho_e = 0 \text{ for } \left\langle 0 < \left|x_{i1}^*\right| < \left|x_{i1}\right| \right\rangle \qquad \textit{Equation 18}$$

Note the difference between p_k and ρ_e. p_k represents the price of market goods for agent j, whereas ρ_e is the price paid by agent j for an improvement in environmental quality and hence corresponds to j's willingness to pay for environmental quality. It follows that ρ_e symbolises how the environment can be transferred into an economic realm. The victim-pays-principle holds when property rights are held by polluters. The situation can be reversed if property rights are reallocated and it can be shown that polluters could then be willing to pay for pollution abatement. This issue will be further examined in Lecture 2 when discussing the Coase theorem and property rights.

Although the victim-pays-principle may seem inapplicable to non-theoretical scenarios there have been instances of it being used in practice. A recent example relates to the case of acid rain in Europe. Countries that were generally not the main emitters of the pollutants that cause acid rain, such as Scandinavian countries, were nonetheless those that were suffering the most from this phenomenon. This led Sweden to pay Poland to abate its use of coal which was one of the causes of acid rain.

SUMMARY

This lecture has aimed to stress the importance of taking into account externalities in economic transactions. The situation where victims of pollution pay polluters to abate was underlined and illustrated mathematically. The important feature of this situation is that property rights are held by the polluters. The issue of the allocation of property rights will be dealt with more extensively in the next lecture.

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